Résolution de problèmes black-box avec LocalSolver

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Innovation 24 & LocalSolver

www.localsolver.com

Who we are

BOUYGUES

Bouygues, one of the French largest corporation, €33 bn in revenues
http://www.bouygues.com

Innovation24

Operations Research subsidiary of Bouygues
20 years of practice and research
http://www.innovation24.fr

LocalSolver

Mathematical optimization solver commercialized by Innovation 24
http://www.localsolver.com

LocalSolver

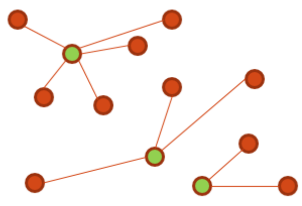
All-terrain optimization solver

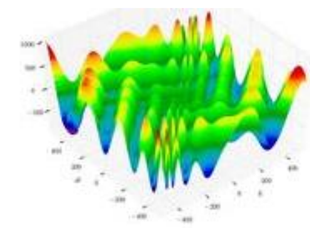
For combinatorial, numerical,
or mixed-variable optimization

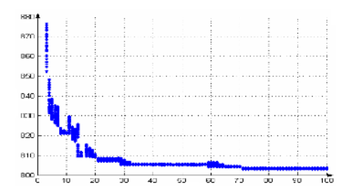
Suited for tackling
large-scale problems

Quality solutions in minutes
without tuning

The « Swiss Army Knife » of mathematical optimization







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P-median

Select a subset P among N points minimizing the sum of distances from each point in N to the nearest point in P

```
function model() {
  x[1..N] <- bool() ; // decisions: point i belongs to P if x[i] = 1
  constraint sum[i in 1..N]( x[i] ) == P ; // constraint: P points selected among N
  minDist[i in 1..N] <- min[j in 1..N]( x[j] ? Dist[i][j] : InfiniteDist ) ; // expressions: distance to the nearest point in P
  minimize sum[i in 1..N]( minDist[i] ) ; // objective: to minimize the sum of distances
}
```

Nothing else to write: “model & run” approach

- Straightforward, natural mathematical model
- Direct resolution: no tuning

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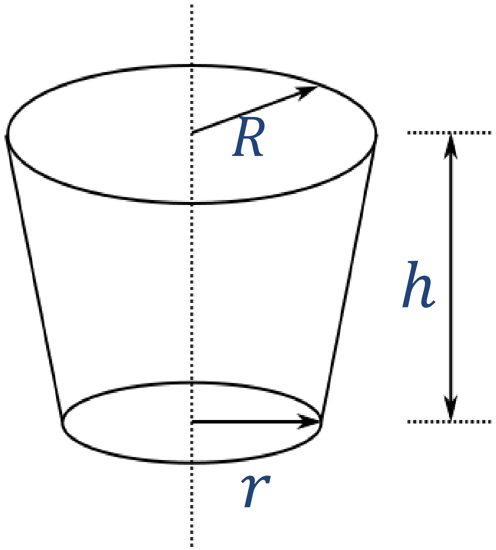
Parametric optimization

Maximize the volume of a bucket with a given surface of metal



$$V = \frac{\pi h}{3}(R^2 + Rr + r^2)$$

$$S = \pi r^2 + \pi(R + r)\sqrt{(R - r)^2 + h^2}$$

```
function model() {
  R <- float(0,1);
  r <- float(0,1);
  h <- float(0,1);

  V <- PI * h / 3.0 * (R*R + R*r + r*r);
  S <- PI * r * r + PI*(R+r) * sqrt(pow(R-r,2) + h*h);

  constraint S <= 1;
  maximize V;
}
```

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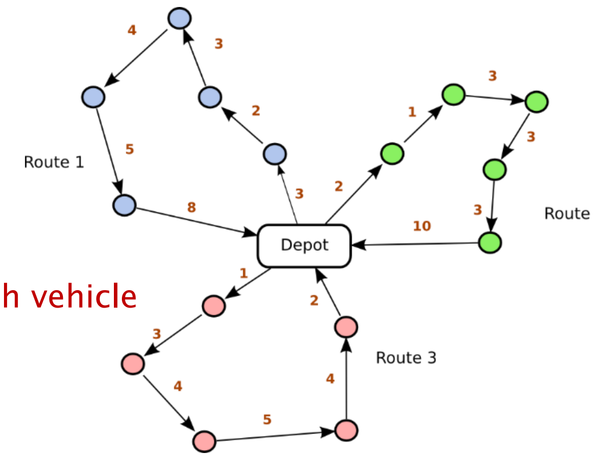
Vehicle routing

Find the shortest set of routes for a fleet of K vehicles in order to deliver to a given set of N customers

```
function model() {
  x[1..K] <- list(N) ; // for each vehicle, order the clients to visit
  constraint partition( x[1..K] ); // each client is visited once
  distances[k in 1..K] <- sum[i in 1..N-1]( dist( x[k][i-1], x[k][i]) )
                                        + dist( x[k][N-1], x[k][0] ); // traveled distance for each vehicle
  minimize sum[k in 1..K]( distances[k] ); // minimize total traveled distance
}
```



Mathematical operators

Decisional	Arithmetical			Logical	Relational	Set-related
bool	sum	sub	prod	not	eq	count
float	min	max	abs	and	neq	indexof
int	div	mod	sqrt	or	geq	partition
list	log	exp	pow	xor	leq	disjoint
	cos	sin	tan	iif	gt	
	floor	ceil	round	array+at	lt	
	dist	scalar		piecewise		

+ operator **call** : to call an external native function
which can be used to implement your own (black-box) operator

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Black-box optimization

Black-box optimization

$$\min f(x)$$
$$x \in [x^L, x^U]$$

Context

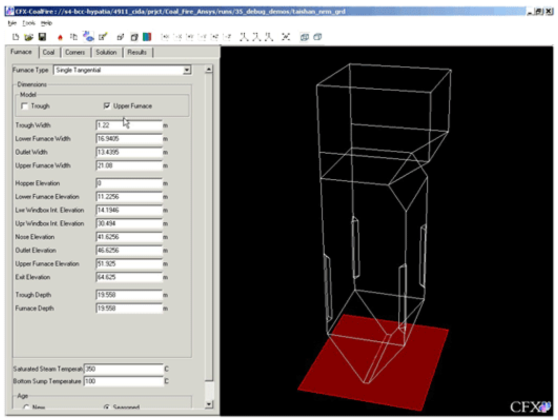
- Function *f* without analytical form (external code or library)
- Maybe be costly to evaluate (minutes or even hours)
- *f* may be non-smooth, noisy, or even non-deterministic
- *f* defined over continuous, integer, boolean variables
- With box constraints (bounds on variables)

Engineering applications

- Product or system design -> parametric optimization
- Simulation optimization

Case study

Designing pulverized coal boilers



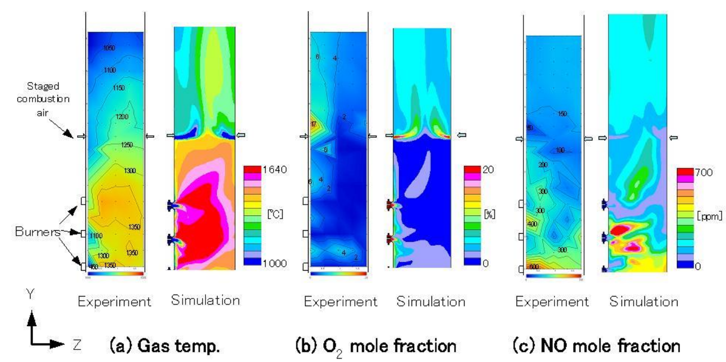
Context

- Optimize coal injection in the boiler
- 6 criteria to optimize: 3 related to energetic performance, 3 related to pollutants
- Each call to the CFD simulator takes 12 hours

Heterogeneous variables

- On/off: boolean variables
- Flow rates: continuous variables
- Angles: discrete variables

→ 3 * 8 = 24 variables to optimize



Designing sailboat weathervanes

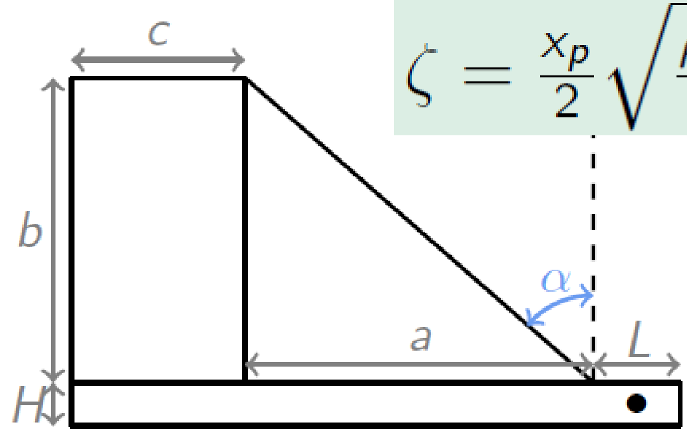
Context

- Used to measure the wind, to drive sailboats
- No CFD simulator but complex analytical formulas to described the physics of the weathervane

Heterogeneous variables

- 4 continuous variables
- Precision: 0.1 millimeters
- Highly nonlinear constraints

$$\zeta = \frac{x_p}{2}\sqrt{\frac{\rho S x_p K}{2I}}$$



Resolution method

Iterative algorithm

1. Build a model of the black-box objective
2. Use the model to find new points to evaluate
   - Minimizing the objective (Intensification)
   - Covering the search space (Diversification)
3. Call the black-box on the new point

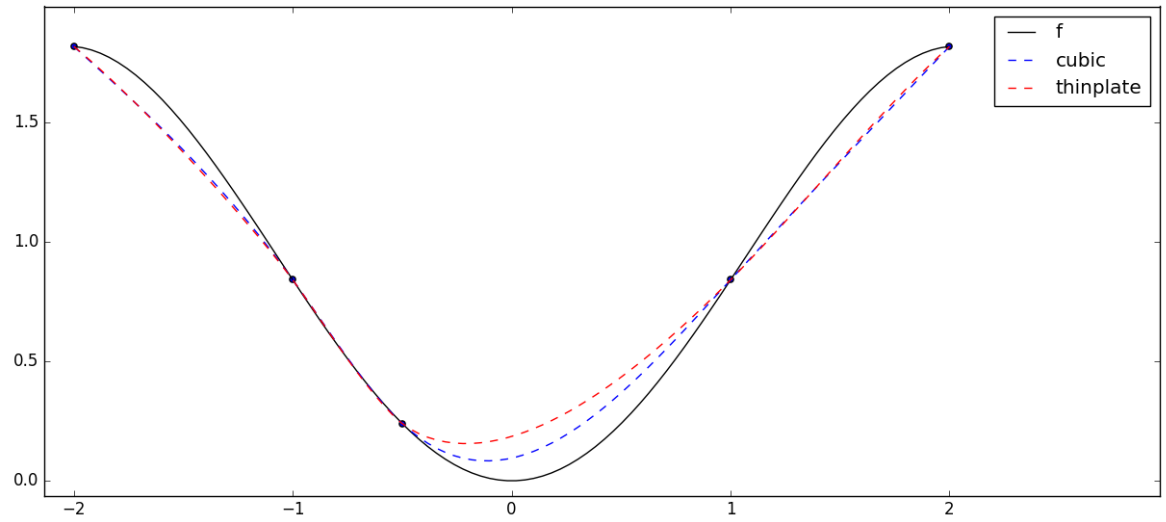
LocalSolver

Surrogate model

Interpolation of evaluated points

- *Radial Basis Function* $\phi(\|x - c\|)$ **[Gutmann 01, Costa et Nannicini 14]**
- Model $m(x) = \sum_{c \in C} \lambda_c \, \phi(\|x - c\|) + p(x)$
- Calculation of parameters λ_c and coefficients in p
  - Interpolation through linear algebra direct techniques
  - Iterative resolution through LocalSolver if ill-conditioned

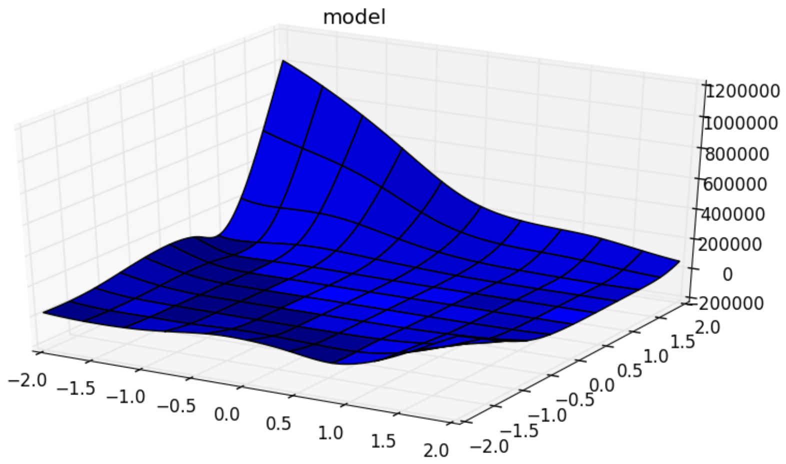
	$\phi(r)$
Cubic	r^3
Gaussian	$e^{-\gamma r}$
Multiquadric	$\sqrt{r^2 + \gamma^2}$
Thinplate	$r^2 \log(r)$



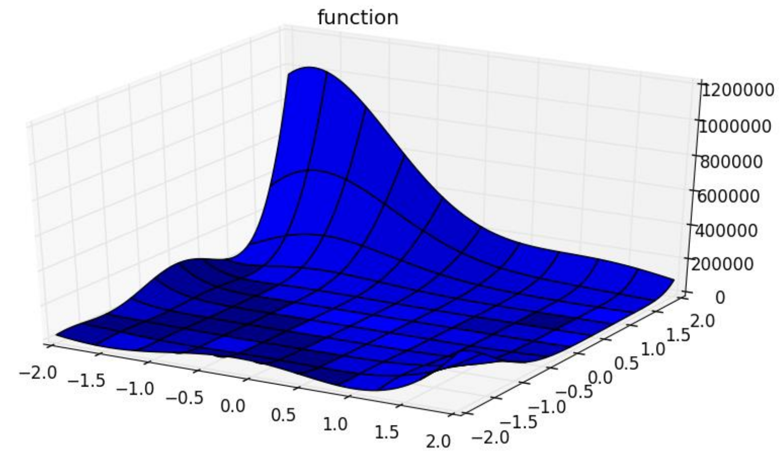
Surrogate model

Improved surrogate model through cross-validation

- Leave-one-out strategy to compute several models
- Select the model minimizing Root-Mean-Square deviation



RBF surrogate model



Real-world objective function

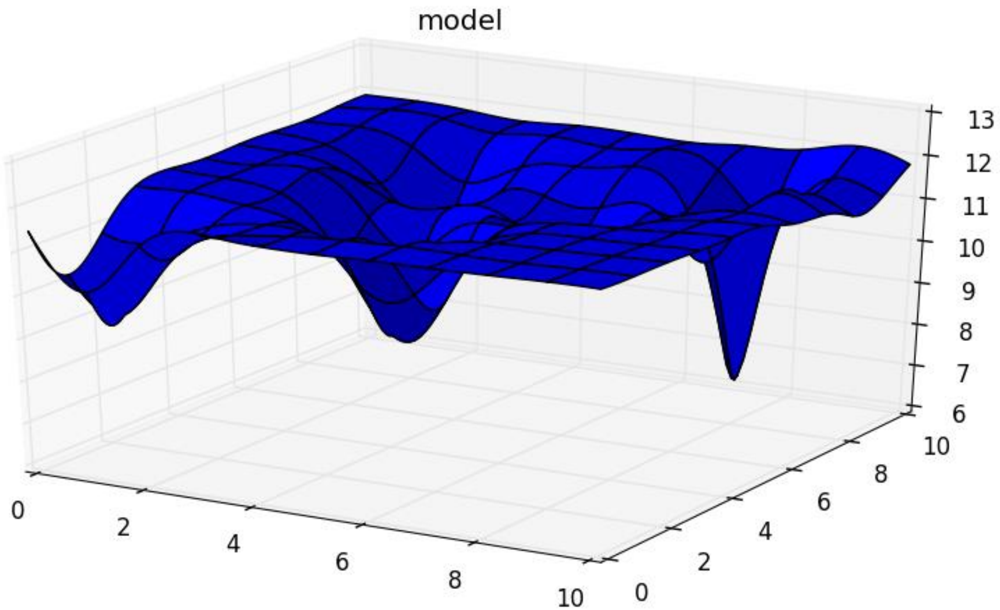
Model-based search

Intensification

- Search for the minimum for x over the surrogate model

$$m(x) = \sum_{c \in C} \lambda_c \, \phi(\|x - c\|) + p(x)$$

- Non-convex optimization over continuous and discrete variables
- Global solution approach through LocalSolver



Model-based search

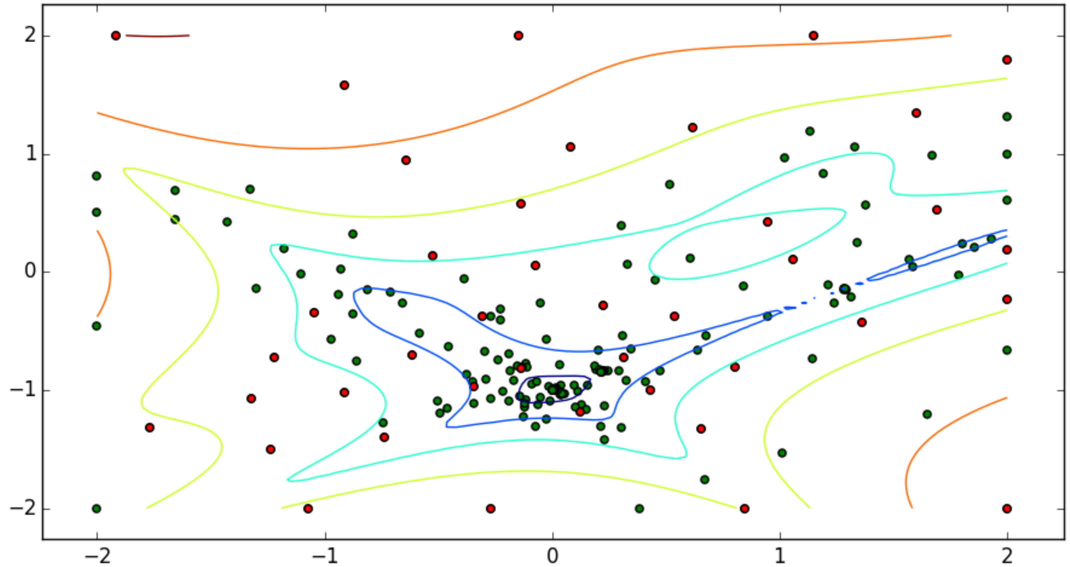
Diversification

- Resolution of a weighted « hostile brother » problem

$$\max\left\{ w(x) * \min_{c} \|x - x_c\| \right\}$$

Penalizes « far-away » points for m — Moves away from known points

- Non-convex optimization over continuous and discrete variables
- Global solution approach through LocalSolver

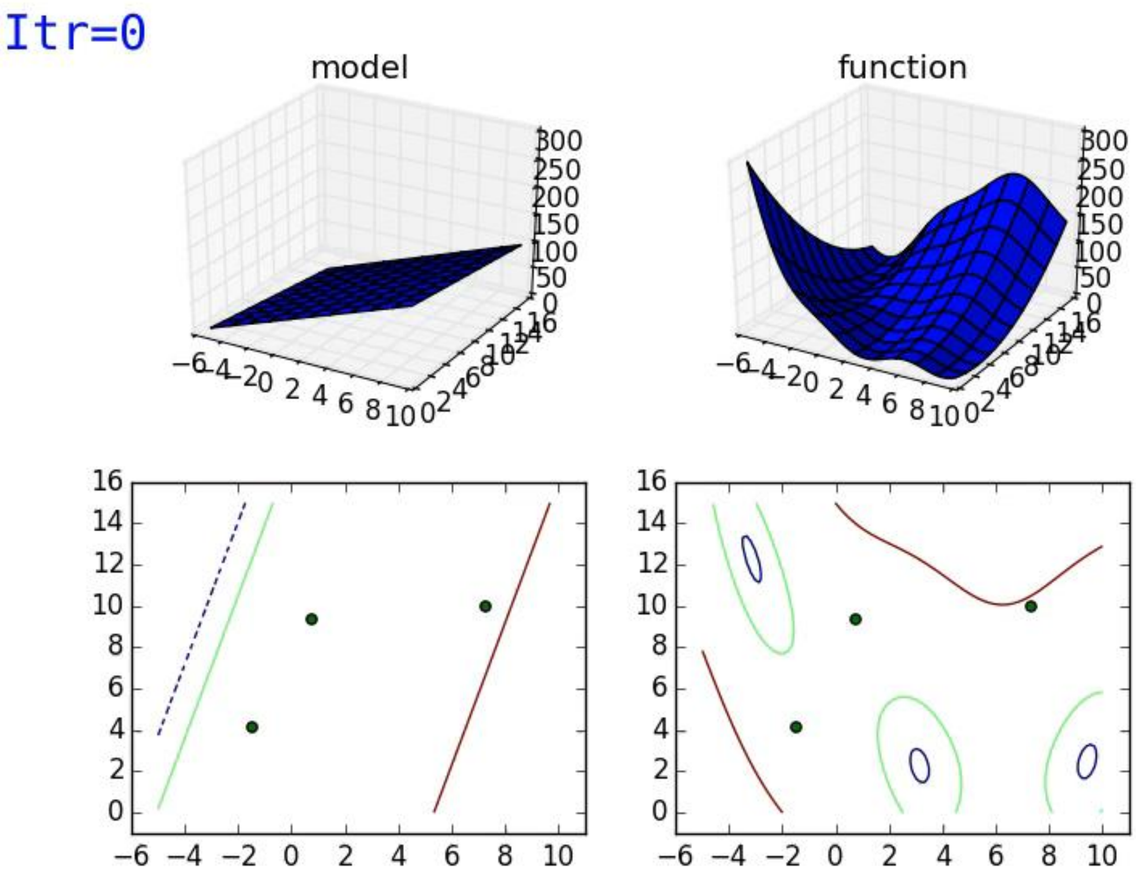


Example

Branin function

$$\min \left(y - \frac{5.1}{4\pi^2}x^2 + \frac{5}{\pi}x - 6\right)^2 + 10 * \left(1 - \frac{1}{8\pi}\right)\cos x + 10$$

$$x \in [-5, 10] \quad y \in [0, 15]$$



Benchmark

Instances

- 25 instances from the most recent paper about black-box optimization
- 20 runs per instance, 150 calls max. to the black-box per run
- Numerical precision: 1e-6

Results

- LocalSolver: 310 opt. solutions found, 94 calls avg. per run
- NOMAD (GERAD): 170 opt. solutions found

Benchmark

	LocalSolver			NOMAD	
Instance	**#sol**	**Avg. Eval**	**Error (%)**	**#sol**	**Error (%)**
branin	20	23	0,0	20	0,0
camel	20	26	0,0	19	4,0
ex_4_1_1	20	11	0,0	20	0,0
ex_4_1_2	20	51	0,0	20	0,0
ex_8_1_1	20	10	0,0	19	2,5
ex_8_1_4	20	44	0,0	0	341,5
gear	20	34	0,0	0	388,0
goldsteinprice	18	122	0,0	16	450,0
hartman3	8	130	0,0	15	9,4
hartman6	8	121	5,1	0	5,7
least	0	150	204,7	0	129,0
nvs04	20	70	194,4	4	9997,0
nvs06	16	127	13,3	9	8,7
nvs09	20	15	0,0	16	1,2
nvs16	8	138	0,0	9	885,0
perm0_8	0	150	147,2	0	412,0
perm_6	0	150	44134,7	0	311032,0
rbrock	20	83	10,8	0	43,2
schoen_10_1	4	145	28,8	0	119,5
schoen_10_2	0	150	1,6	0	115,7
schoen_6_1	18	101	1,8	0	51,5
schoen_6_2	10	120	32,7	0	54,2
shekel10	8	118	60,1	0	56,9
shekel5	6	127	51,7	1	46,1
shekel7	6	127	47,0	2	47,9
	310			170	

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