Solving routing & scheduling problems through set-based modeling

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Innovation 24 & LocalSolver

www.localsolver.com

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Who we are

Bouygues, one of the French largest corporation, €33 bn in revenues
http://www.bouygues.com

Operations Research subsidiary of Bouygues
20 years of practice and research
http://www.innovation24.fr

Mathematical optimization solver developed by Innovation 24
http://www.localsolver.com

LocalSolver

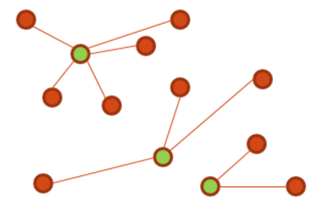
All-terrain optimization solver

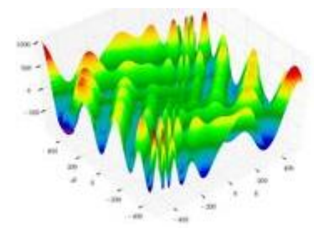
For combinatorial, numerical,
or mixed-variable optimization

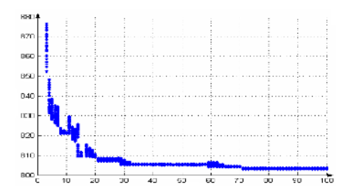
Suited for tackling
large-scale problems

Quality solutions in minutes
without tuning

The « Swiss Army Knife » of
mathematical optimization







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www.localsolver.com

Clients

- Construction
- Medias & Advertising
- Telco & Retail
- Large Industry
- Energy
- Banking & Finance
- Transportation
- Logistics
- Food & Agribusiness
- Aerospace & Defense
- IT Services

LocalSolver

Quick tour

Features

Better solutions faster

- Provides high-quality solutions quickly (minutes)
- Scalable: able to tackle problems with millions of decisions

Easy to use

- « Model & Run »
  - Rich but simple mathematical modeling formalism
  - Direct resolution: no need of complex tuning

- Innovative modeling language for fast prototyping
- Object-oriented C++, Java, .NET, Python APIs for tight integration
- Fully portable: Windows, Linux, Mac OS (x86, x64)

P-median

Select a subset P among N points minimizing the sum of distances from each point in N to the nearest point in P

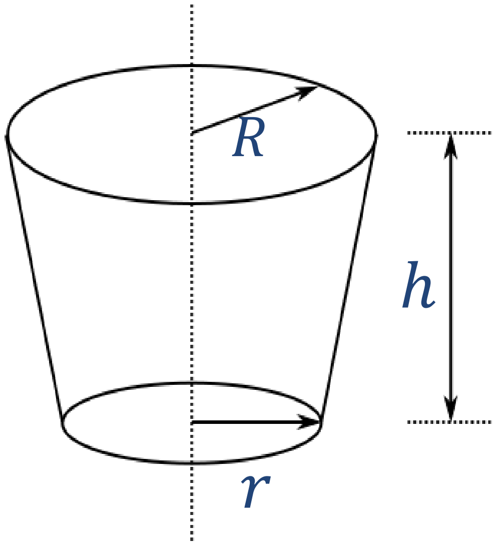
```
function model() {
  x[1..N] <- bool() ; // decisions: point i belongs to P if x[i] = 1
  constraint sum[i in 1..N]( x[i] ) == P ; // constraint: P points selected among N
  minDist[i in 1..N] <- min[j in 1..N]( x[j] ? Dist[i][j] : InfiniteDist ) ; // expressions: distance to the nearest point in P
  minimize sum[i in 1..N]( minDist[i] ) ; // objective: to minimize the sum of distances
}
```

Nothing else to write: “model & run” approach

- Straightforward, natural mathematical model
- Direct resolution: no tuning

Parametric optimization

Maximize the volume of a bucket with a given surface of metal



$$V = \frac{\pi h}{3}\left(R^2 + Rr + r^2\right)$$

$$S = \pi r^2 + \pi(R+r)\sqrt{(R-r)^2 + h^2}$$

```
function model() {

  R <- float(0,1);
  r <- float(0,1);
  h <- float(0,1);

  V <- PI * h / 3.0 * (R*R + R*r + r*r);
  S <- PI * r * r + PI*(R+r) * sqrt(pow(R-r,2) + h*h);

  constraint S <= 1;
  maximize V;
}
```

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Mathematical operators

Decisional	Arithmetical			Logical	Relational	Set-related
bool	sum	sub	prod	not	eq	count
float	min	max	abs	and	neq	at
int	div	mod	sqrt	or	geq	indexof
list	log	exp	pow	xor	leq	partition
	cos	sin	tan	iif	gt	disjoint
	floor	ceil	round	array + at	lt	
	dist	scalar		piecewise		

+ operator call : to call an external native function
which can be used to implement your own (black-box) operator

Smart APIs

C++ ISO

Java 5.0

.NET C# 2.0

Python 2.7, 3.2, 3.4

```
########## optimal_bucket.py ##########

import localsolver
import sys

with localsolver.LocalSolver() as ls:

    PI = 3.14159265359

    #
    # Declares the optimization model
    #
    m = ls.model

    R = m.float(0,1)
    r = m.float(0,1)
    h = m.float(0,1)

    # Surface constraint
    # surface = PI * r^2 + PI*(R+r) * sqrt ((R-r)^2 + h^2)
    surface = PI*r*r + PI * m.sqrt((R-r)**2 + h**2) * (R+r)
    m.constraint(surface <= PI)

    # Maximize volume
    # volume = PI * h/3 * (R^2 + R*r + r^2)
    volume = PI * h/3 * (R**2+ R*r + r**2)
    m.maximize(volume)

    m.close()

    #
    # Param
    #
    ls.param.nb_threads = 2
    if len(sys.argv) >= 3: ls.create_phase().time_limit = int(sys.argv[2])
    else: ls.create_phase().time_limit = 6

    ls.solve()
```

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Motivations

Modeling approaches for
the Traveling Salesman Problem

Natural modeling

As a permutation

In Rosen: "Permutations and Combinations"

The Traveling Salesman Problem (TSP)

TSP: Given a list of cities and their pairwise distances, find a shortest possible tour that visits each city exactly once.

Objective: find a permutation $a_1, \ldots, a_n$ of the cities that minimizes

$$d(a_1, a_2) + d(a_2, a_3) + \ldots + d(a_{n-1}, a_n) + d(a_n, a_1)$$

where $d(i, j)$ is the distance between cities i and j

An optimal TSP tour through Germany's 15 largest cities

Reference modeling

In Garey & Johnson: "Computers and Intractability"

[ND22] TRAVELING SALESMAN

INSTANCE: Set C of m cities, distance $d(c_i, c_j) \in Z^+$ for each pair of cities $c_i, c_j \in C$, positive integer B.

QUESTION: Is there a tour of C having length B or less, i.e., a permutation $< c_{\pi(1)}, c_{\pi(2)}, \ldots, c_{\pi(m)} >$ of C such that

$$\left[\sum_{i=1}^{m-1} d(c_{\pi(i)}, c_{\pi(i+1)}) \right] + d(c_{\pi(m)}, c_{\pi(1)}) \leqslant B \ ?$$

Mixed-Integer Linear Programming

In Orman & Williams: "A Survey of Different Integer Programming Formulations of the TSP"

Classical model has an **exponential** number of constraints

Minimise $\sum_{\substack{i,j \\ i \neq j}} c_{ij} x_{ij}$

Conventional Formulation (C) (Dantzig, Fulkerson and Johnson (1954))

$$\sum_{\substack{j \\ j \neq i}} x_{ij} = 1 \qquad \forall\ i \in N$$

$$\sum_{\substack{i \\ i \neq j}} x_{ij} = 1 \qquad \forall\ j \in N$$

$$\sum_{\substack{i,j \in M \\ i \neq j}} x_{ij} \leq |M| - 1 \qquad \forall\ M \subset N \text{ such that } \{1\} \notin M, |M| \geq 2$$

→ Iterative procedure to generate subtour elimination constraints

Set-based modeling

Innovative modeling concepts
for routing & scheduling problems

List variables

Structured multi-valued decisional operator list(n)

- Order a subset of values in domain {0, …, n-1}
- Each value is unique in the list

Classical operators to interact with “list”

- count(u): number of values selected in the list
- at(u,i) or u[i]: value at index i in the list
- indexOf(u,v): index of value v in the list
- disjoint(u1, u2, …, uk): true if u1, u2, …, uk are pairwise disjoint
- partition(u1, u2, …, uk): true if u1, u2, …, uk induce a partition of {0, …, n-1}

Traveling salesman

```
function model() {
  x <- list(N) ; // order n cities {0, ..., n-1} to visit
  constraint count(x) == N; // exactly n cities to visit
  minimize sum[i in 1..N-1]( Dist[ x[i-1] ][ x[i] ] )
                  + Dist[ x[N-1] ][ x[0] ] ; // minimize sum of traveled distances
}
```

TSP: Given a list of cities and their pairwise distances, find a shortest possible tour that visits each city exactly once.

Objective: find a permutation $a_1, \ldots, a_n$ of the cities that minimizes

$$d(a_1, a_2) + d(a_2, a_3) + \ldots + d(a_{n-1}, a_n) + d(a_n, a_1)$$

where $d(i, j)$ is the distance between cities i and j

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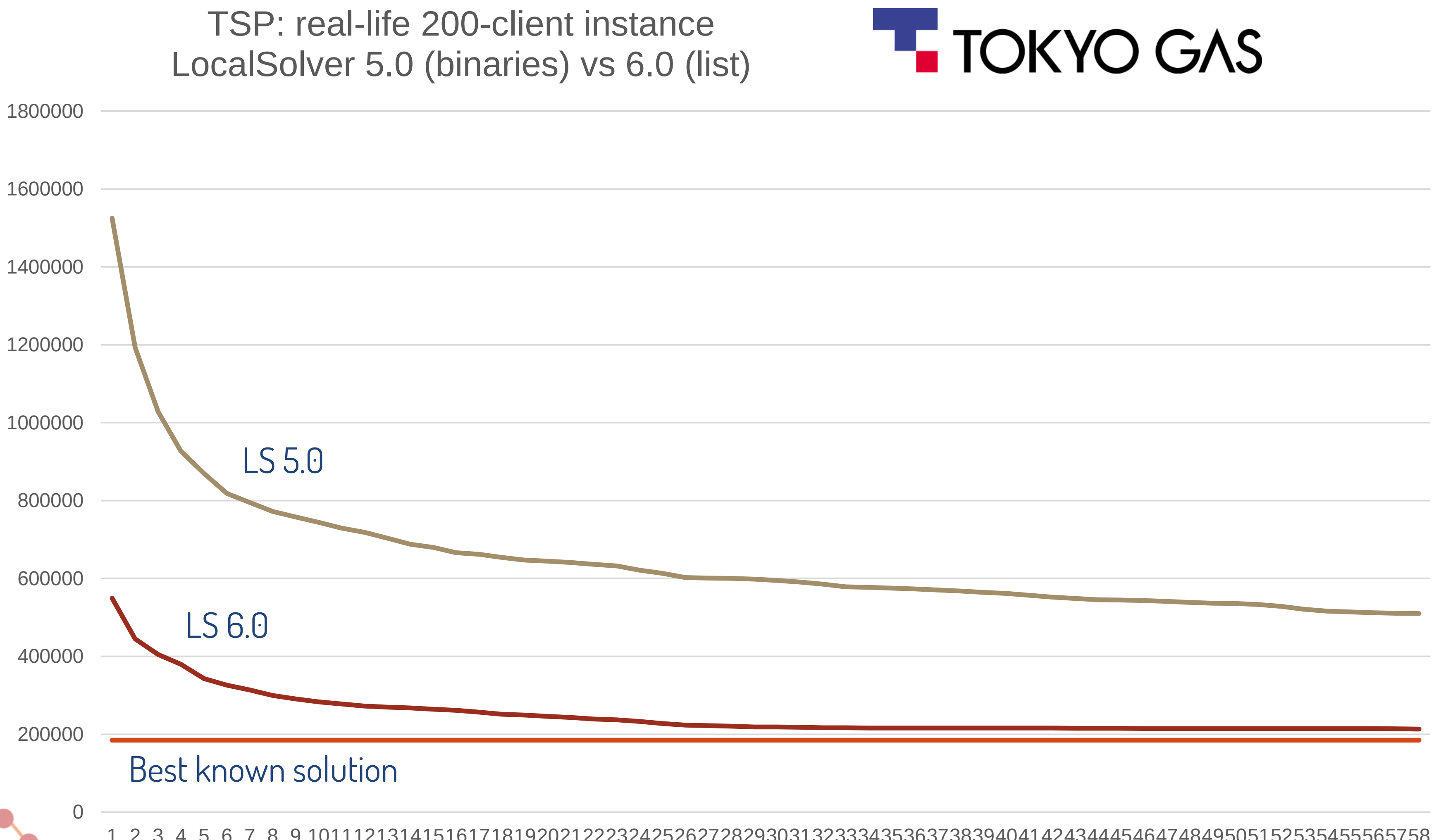
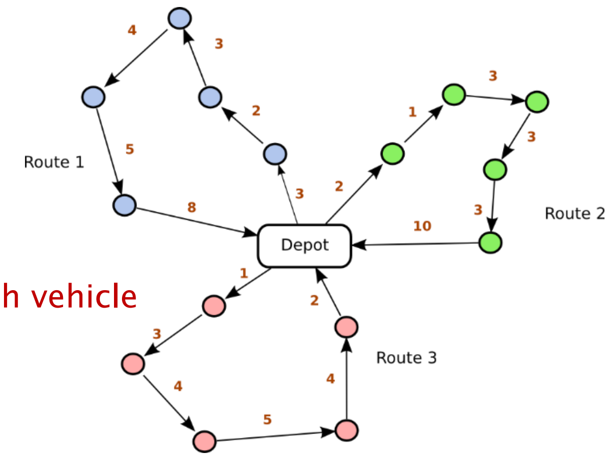
Performance



Vehicle routing

Find the shortest set of routes for a fleet of K vehicles in order to deliver to a given set of N customers

```
function model() {
  x[1..K] <- list(N) ; // for each vehicle, order the clients to visit
  constraint partition( x[1..K] ); // each client is visited once
  distances[k in 1..K] <- sum[i in 1..N-1]( dist( x[k][i-1], x[k][i]) )
                                   + dist( x[k][N-1], x[k][0] ); // traveled distance for each vehicle
  minimize sum[k in 1..K]( distances[k] ); // minimize total traveled distance
}
```



CVRPTW benchmarks

CVRPTW – instances Solomon R100

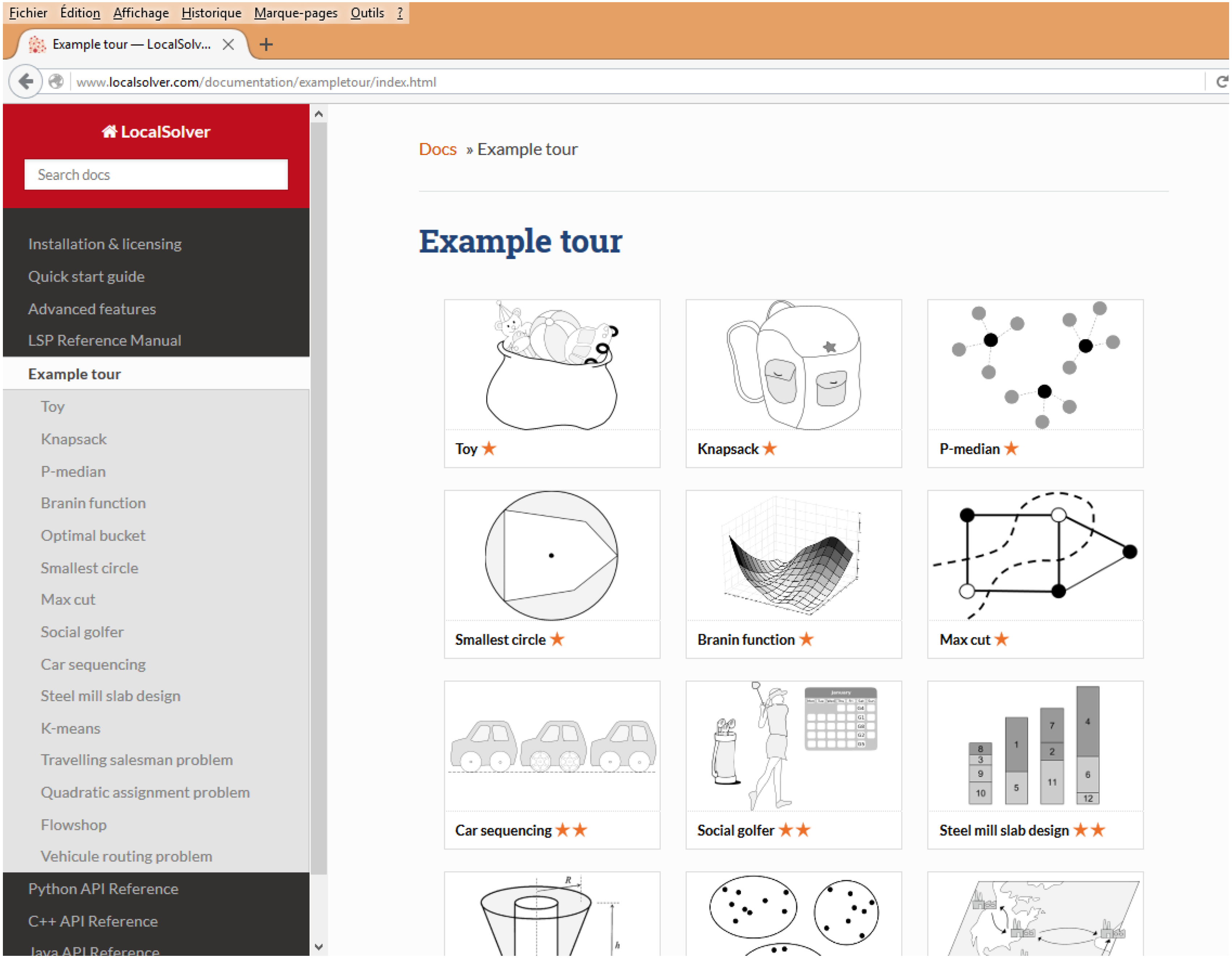
- 101 to 112 clients, 19 trucks max.
- 13 instances
- 5 minutes of running time
- LS list: 3 % avg. opt. gap

CVRPTW – instances Solomon R200

- 201 to 208 clients, 4 trucks max.
- 8 instances
- 5 minutes of running time
- LS list: 8 % avg. opt. gap

Beyond routing problems

Planning, scheduling, sequencing



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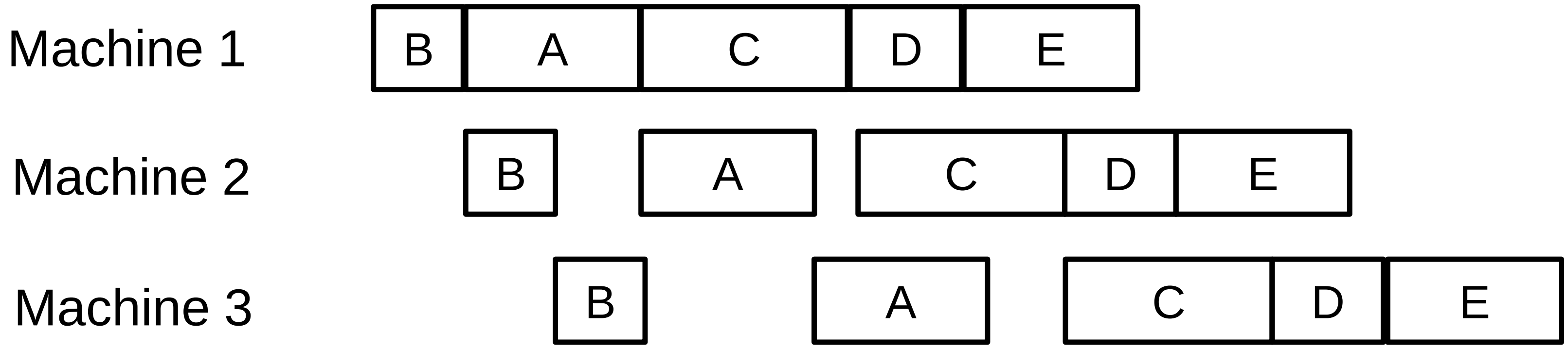
Steel mill slab design ★★

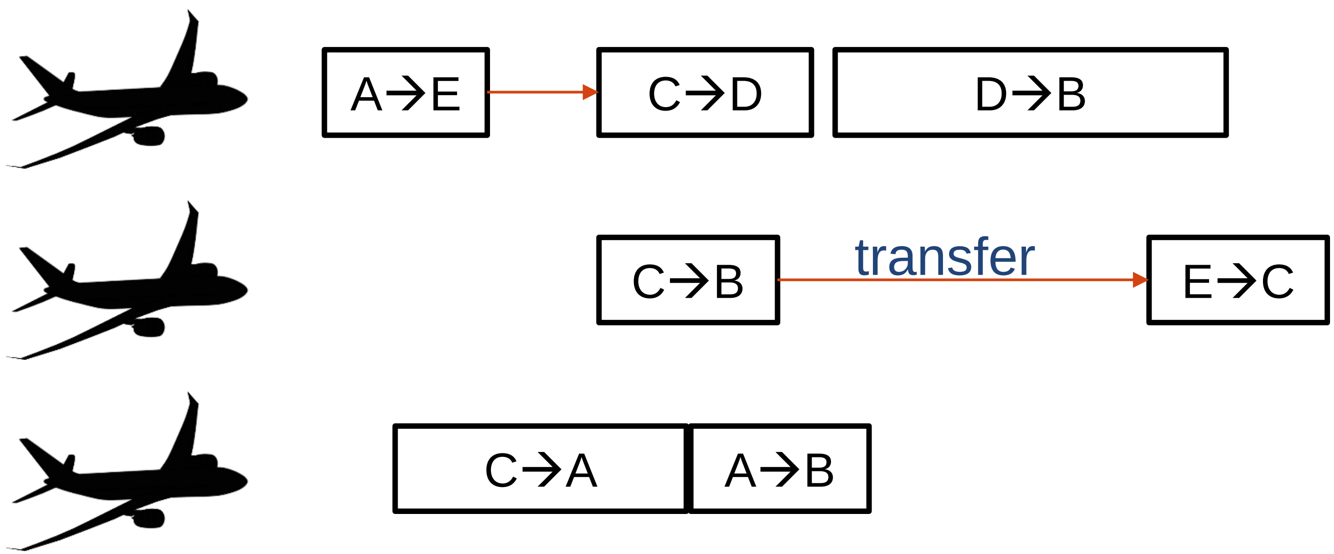
Flow-shop scheduling



Since we are looking for a permutation of jobs,
the model is straightforward with a single list variable

Planning

To assign flights to planes



A solution is a partition of flights into K lists (one per plane)

The goal is to minimize the total transfer times

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Conclusion

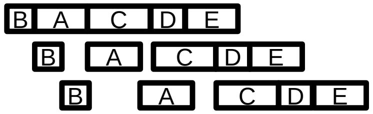
Set-based modeling

List variables = first step towards set-based modeling

This higher level of modeling yields simple and compact models producing high-quality solutions for many kinds of problems

Routing

Scheduling



Planning

STELLAR

RCPSP

Leibniz Universität Hannover

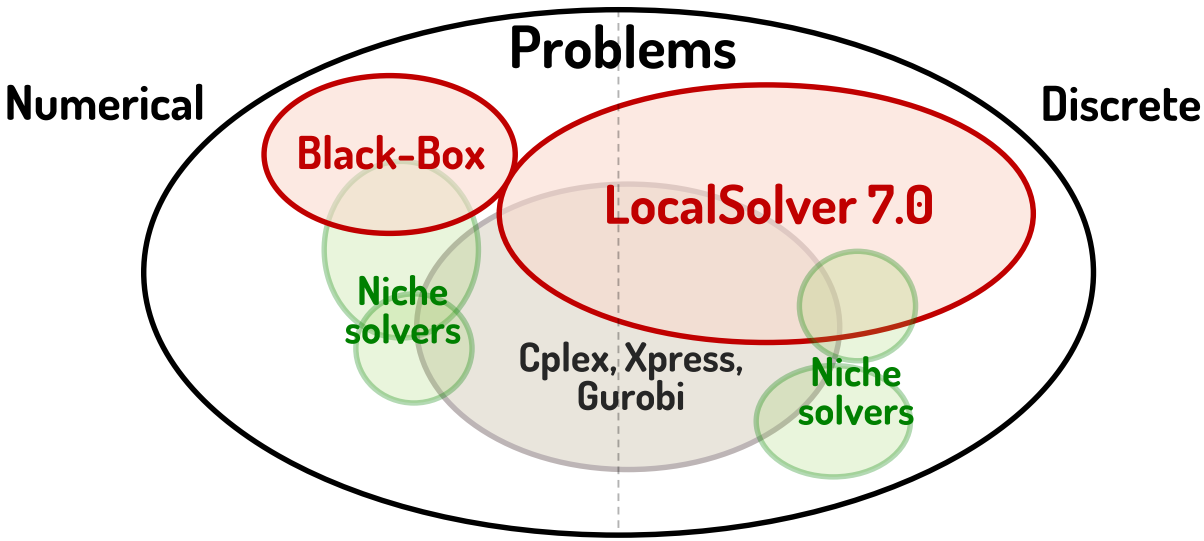
Any sequencing problem

LocalSolver 7.0, December 2016

Major features

- Integration of the power of LP/MIP techniques into LocalSolver
- Improved performance for set-based models
- Improved performance for numerical (nonlinear) models

→ First ingredients in 6.5 version planned for July 2016



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