



LocalSolver
mathematical programming by local search

www.localsolver.com

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LocalSolver

A solver aligned with enterprise needs

- Handle highly combinatorial problems
- Solve problems with millions of decisions
- Provides high-quality solutions in seconds
- Proves optimality when possible (best effort)

A solver aligned with practitioner needs

- « Model & Run »
 - Simple mathematical modeling formalism
 - Direct resolution: no need of complex tuning
- A simple and transparent pricing



LocalSolver

New-generation solver

- Computing good-quality solutions by local search
- Scalable (each iteration done in sublinear time)
- Able to optimize in nonconvex and nonsmooth spaces
- Handle multiple nonlinear objective functions
- Computing lower bounds separately (relaxation, inference, cuts)

Portable software

- An innovative modeling language for fast prototyping
- Light object-oriented APIs: C++, Java, .NET
- Fully portable: Windows, Linux, Mac OS (x86, x64)



LocalSolver technology

Autonomous local search

- Generic moves based on ejection chains
- Learning of effective moves
- Sublinear evaluation of moves
- Easy to tune simulated annealing

Efficient implementation in C++

- Multithreaded search
- Incremental algorithmic
- Memory management



LocalSolver

Quick tour



Quadratic Knapsack

8 items to pack in a sack: maximize the total value of items while not exceeding a total weight of 102 kg

```
function model() {  
    // 0-1 decisions  
    x_0 <- bool(); x_1 <- bool(); x_2 <- bool(); x_3 <- bool();  
    x_4 <- bool(); x_5 <- bool(); x_6 <- bool(); x_7 <- bool();  
  
    // weight constraint  
    knapsackWeight <- 10*x_0+ 60*x_1+ 30*x_2+ 40*x_3+ 30*x_4+ 20*x_5+ 20*x_6+ 2*x_7;  
    constraint knapsackWeight <= 102;  
  
    // maximize value  
    knapsackValue <- 1*x_0+ 10*x_1+ 15*x_2+ 40*x_3+ 60*x_4+ 90*x_5+ 100*x_6+ 15*x_7  
        +150 * x_0 * x_1 + 200 *x_3*x_4;  
    maximize knapsackValue;  
}
```

Binary decision variables

Integer intermediate variables

Quadratic expressions

The user writes the model: nothing else to do!
declarative approach = model & run



Multiobjective knapsack

```
function model() {  
    // 0-1 decisions  
    x[0..7] <- bool();  
  
    // weight constraint  
    knapsackWeight <- 10*x[0]+ 60*x[1]+ 30*x[2]+ 40*x[3]+ 30*x[4]+ 20*x[5]+ 20*x[6]+ 2*x[7];  
    constraint knapsackWeight <= 102;  
  
    // maximize value  
    knapsackValue <- 1*x[0]+ 10*x[1]+ 15*x[2]+ 40*x[3]+ 60*x[4]+ 90*x[5]+ 100*x[6]+ 15*x[7]  
        + 150 * x[0] * x[1] + 200 *x[3]*x[4];  
    maximize knapsackValue;  
  
    // secondary objective: minimize product of minimum and maximum values  
    knapsackMinValue <- min[i in 0..7](x[i] ? values[i] : infinity);  
    knapsackMaxValue <- max[i in 0..7](x[i] ? values[i] : 0);  
    knapsackProduct <- knapsackMinValue * knapsackMaxValue;  
    minimize knapsackProduct;  
}
```

Nonlinear operators: prod, min, max,
and, or, if-then-else, ...

Lexicographic objectives



Mathematical operators

Arithmetic		Logical	Relational
sum	prod	not	==
min	max	and	!=
div	mod	or	<=
abs	dist	xor	>=
sqrt		if	$<$
			$>$



Modeling APIs

```
function model() {  
    // 0-1 decisions  
    x[1..nbItems]  
  
    // weight constraint  
    knapsackWeight <- 10*x0 + 60*x1 + 30*x2 + 40*x3 + 30*x4 + 20*x5 + 20*x6 + 2*x7;  
    constraint knapsackWeight <= 102;  
  
    // maximize knapsackValue  
    knapsackValue <- 1*x0 + 10*x1 + 15*x2 + 40*x3 + 60*x4 + 90*x5 + 100*x6 + 15*x7;  
    maximize knapsackValue;  
}
```

```
#include "localsolver.h"  
using namespace localsolver;  
  
int main()  
{  
    int nbItems = 8;  
    int knapsackWeight = 102;  
    int knapsackValue = 0;  
  
    // LocalSolver API  
    LSEngine localsolver;  
    for (int i = 0; i < nbItems; i++)  
    {  
        LSEVariable x = localsolver.CreateVariable("x" + i);  
        x.SetType(LSVariableType.Bool);  
        x.SetDomain(0, 1);  
    }  
  
    LSModel model = localsolver.GetModel();  
  
    // 0-1 decisions  
    LSEExpression[] x = new LSEExpression[8];  
    for (int i = 0; i < 8; i++)  
        x[i] = model.CreateExpression(LSOperator.Bool);  
  
    // knapsackWeight <- 10*x0 + 60*x1 + 30*x2 + 40*x3 + 30*x4 + 20*x5 + 20*x6 + 2*x7;  
    LSEExpression knapsackWeight = model.CreateExpression(LSOperator.Sum);  
    for (int i = 0; i < 8; i++)  
        knapsackWeight.AddOperand(model.CreateExpression(LSOperator.Prod, weights[i], x[i]));  
  
    // knapsackWeight <= 102;  
    model.AddConstraint(model.CreateExpression(LSOperator.LessOrEqual, knapsackWeight, 102));  
  
    // knapsackValue <- 1*x0 + 10*x1 + 15*x2 + 40*x3 + 60*x4 + 90*x5 + 100*x6 + 15*x7;  
    LSEExpression knapsackValue = model.CreateExpression(LSOperator.Sum);  
    for (int i = 0; i < 8; i++)  
        knapsackValue.AddOperand(model.CreateExpression(LSOperator.Prod, values[i], x[i]));  
  
    // maximize knapsackValue;  
    model.AddObjective(knapsackValue, LSObjectiveDirection.Maximize);  
  
    // close the model before solving it  
    model.Close();  
    LSPhase phase = localsolver.CreatePhase();  
    phase.SetTimeLimit(1);  
    localsolver.Solve();  
}
```

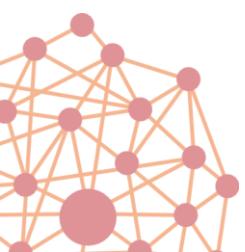
C++

Java

C#

createExpression

addOperand



LocalSolver

Real-life applications



Car sequencing

Scheduling cars along an assembly line

- Each car requires some options
- Each option induces a ratio constraint P/Q
- A class is a set of cars requiring the same options



Objective: to space options over the line

- We wish no more than 2 sunroofs over 5 consecutive cars
- For any window of 5 cars, a penalty is computed as $\max(n-2, 0)$ with n the number of cars requiring a sunroof



LSP model

$x_{cp} = 1 \Leftrightarrow \text{car of class } c \text{ is in position } p$

```
x[1..nbClasses][1..nbPositions] <- bool();  
  
for [c in 1..nbClasses]  
    constraint sum[p in 1..nbPositions](x[c][p]) == card[c];  
  
for [p in 1..nbPositions]  
    constraint sum[c in 1..nbClasses](x[c][p]) == 1;  
  
op[o in 1..nbOptions][p in 1..nbPositions] <- or[c in 1..nbClasses : options[c][o]](x[c][p]);  
  
nbVehicles[o in 1..nbOptions][j in 1..nbPositions-Q[o]+1] <- sum[k in 1..Q[o]](op[o][j+k-1]);  
  
violations[o in 1..nbOptions][j in 1..nbPositions-Q[o]+1] <- max(nbVehicles[o][j] - P[o], 0);  
  
obj <- sum[o in 1..nbOptions][p in 1..nbPositions-Q[o]+1](violations[o][p]);  
minimize obj;
```



Car sequencing at Renault

Additional constraints: no more than 10 consecutive cars with the same color



```
color[p in 1..nbPositions] <- sum[c in 1..nbClasses](color[c] * x[c][p]);  
same[p in 2..nbPositions] <- color[p] == color[p-1];  
toolong[p in 11..nbPositions] <- and[j in p-10..p](same[j]);  
for [p in 11..nbPositions] constraint not(toolong[p]);
```

Decision variables
remain the same

Additional objective: minimize the number of paint color changes

```
minimize sum[i in 2..nbPositions](not(same[i]));
```

Problem posed by Renault as 2005 ROADEF Challenge
LocalSolver competitive with finalists (ranked 16/55)



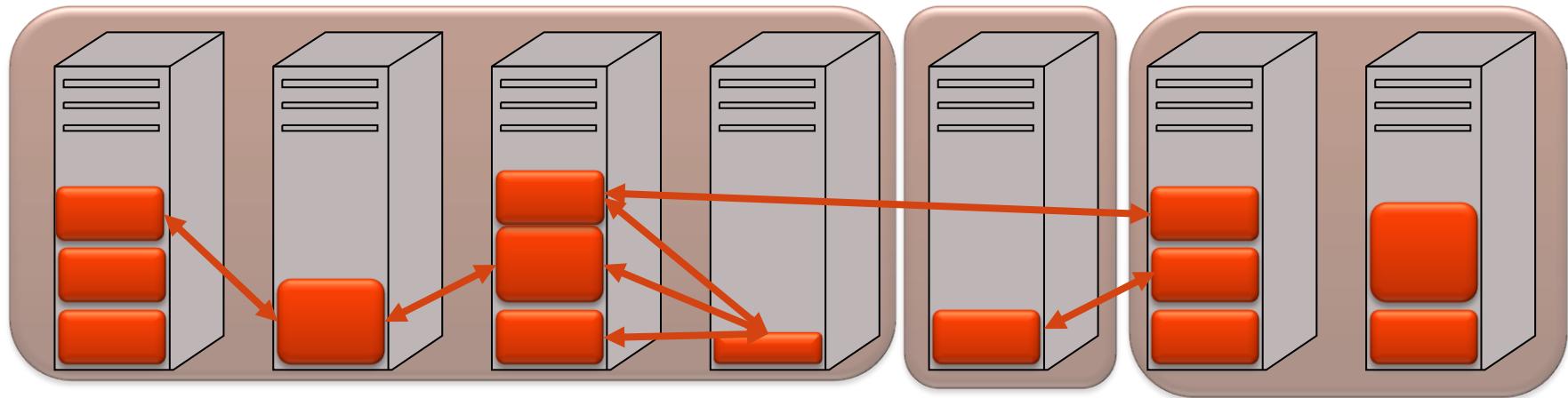
2012 ROADEF/EURO Challenge



EURO



Reassignment of processes to machines, with different kinds of constraints (mutual exclusion, resources, etc.)



More than 100 000 binary decisions

Only 1 day of work

LocalSolver in the final round (Challenge stream)

Roadmap

LocalSolver 2.1 (Released in July 2012)

- Support of arrays
- Memory & speed improvements

LocalSolver 2.2 (October 2012)

- Floating coefficients
- More nonlinear operators (exp, log, cos, sin, tan)
- New autonomous moves

LocalSolver 3 (current 2013)

- Continuous decision variables
- Lower bounds by relaxation



For more details



T. Benoist, B. Estellon, F. Gardi, R. Megel, K. Nouioua.
LocalSolver 1.x: a black-box local-search solver for 0-1 programming.
4OR, A Quarterly Journal of Operations Research 9(3), pp. 299-316.

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Meet us on exhibition booth #20



LocalSolver

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