LocalSolver

Set-based modeling
and black-box optimization

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Innovation 24 & LocalSolver

www.localsolver.com

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Who we are

Bouygues, one of the French largest corporation, €33 bn in revenues
http://www.bouygues.com

Operations Research subsidiary of Bouygues
20 years of practice and research
http://www.innovation24.fr

Mathematical optimization solver commercialized by Innovation 24
http://www.localsolver.com

LocalSolver

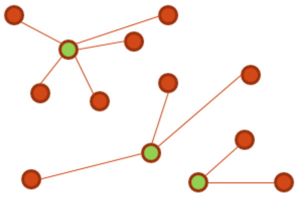
Hybrid optimization solver

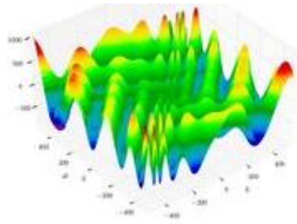
For combinatorial, numerical,
or mixed-variable optimization

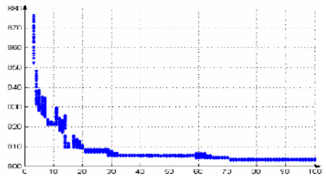
Suited for large-scale
nonlinear optimization

Quality solutions quickly
without tuning

LocalSolver
=
LS + CP/SAT + LP/MIP + NLP







free trial with support – free for academics

www.localsolver.com

LocalSolver origination

Automating local search

Local search

An iterative improvement method

- Explore a neighborhood of the current solution
- Smaller or larger neighborhoods

$\longrightarrow$ Incomplete exploration of the solution space

Essential in combinatorial optimization

- Hidden behind many textbook algorithms (ex: simplex, max flow)
- In the heart of all metaheuristic approaches
- Proved to be inefficient in the worst case
- Largely used because very effective in practice

Why local search?

When it is hopeless to enumerate

- Large-scale combinatorial problems
- When relaxation or inference brings nothing
  (ex: linear relaxation is very fractional)
- When computing relaxation or inference is costly

Adapted to client needs

- Good-quality optima satisfy them
- Fast: each iteration runs in sublinear or even constant time

⟶ Solutions in short running times + ability to scale

Existing tools

Libraries and frameworks

- Complex to handle
- Limited to practitioners having good programming skills
- Don’t address key points (ex: moves)

Solvers integrating “pure” local search

- Pioneering works in SAT community
- MIP & CP: a few attempts but a limited impact (Nonobe & Ibaraki 2001)
- MIP & CP: a lot of heuristic ingredients but no “pure” local search

LocalSolver project

2007: launch

- Define a generic modeling formalism (close to MIP) suited for a local search-based resolution (*model*)
- Develop an effective solver based on pure local search with first principle: “to do what an expert would do” (*run*)

2010: first release

- Large-scale combinatorial problems – especially assignment, packing, covering, partitioning problems – out of scope of classical solvers
- Integration in Innovation 24’s optimization solutions
- First uses outside Innovation 24

LocalSolver today

Quick tour

P-median

Select a subset P among N points minimizing the sum of distances from each point in N to the nearest point in P

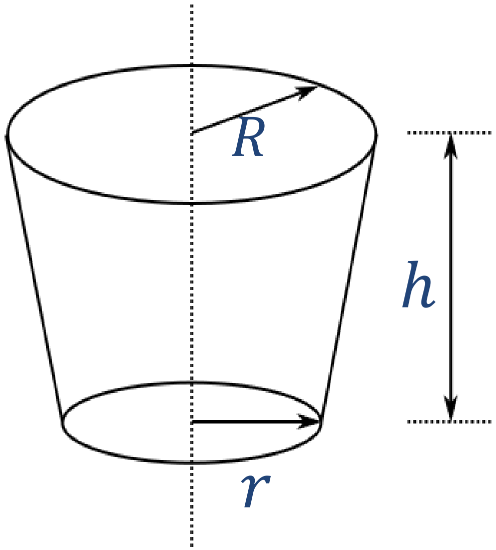
```
function model() {
  x[1..N] <- bool() ; // decisions: point i belongs to P if x[i] = 1
  constraint sum[i in 1..N]( x[i] ) == P ; // constraint: P points selected among N
  minDist[i in 1..N] <- min[j in 1..N]( x[j] ? Dist[i][j] : InfiniteDist ) ; // expressions: distance to the nearest point in P
  minimize sum[i in 1..N]( minDist[i] ) ; // objective: to minimize the sum of distances
}
```

Nothing else to write: "model & run" approach

- Straightforward, natural mathematical model
- Direct resolution: no tuning

Design optimization

Maximize the volume of a bucket with a given surface of metal*



$$V = \frac{\pi h}{3}\left(R^2 + Rr + r^2\right)$$

$$S = \pi r^2 + \pi(R + r)\sqrt{(R - r)^2 + h^2}$$

```
function model() {

  R <- float(0,1);
  r <- float(0,1);
  h <- float(0,1);

  V <- PI * h / 3.0 * (R*R + R*r + r*r);
  S <- PI * r * r + PI*(R+r) * sqrt(pow(R-r,2) + h*h);

  constraint S <= 1;
  maximize V;
}
```

*http://datagenetics.com

Mathematical operators

Decisional	Arithmetic			Logical	Relational
bool	sum	sub	prod	not	==
float	min	max	abs	and	!=
int	div	mod	sqrt	or	<=
list	log	exp	pow	xor	>=
	cos	sin	tan	if-then-else	<
	floor	ceil	round	array + at	>
				piecewise	

Car sequencing

2005 ROADEF Challenge: http://challenge.roadef.org/2005/en

Large-scale instances

- Until 1,300 vehicles to sequence: 400,000 binary decisions

Instance with only 540 vehicles

- Small instance: 44,000 binary decisions
- State of the art: 3,109 (winner of the Challenge)
- Lower bound: 3,103

Minimization

RENAULT

Results

- Gurobi 5.5: 3.027e+06 in 10 min | 194,161 in 1 hour
- LocalSolver 5.0: 3,140 in 10 sec | 3,113 in 10 min

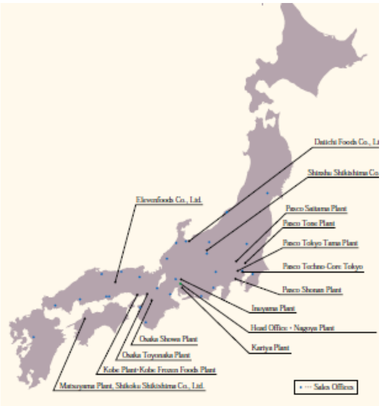
Supply chain optimization

Global supply chain optimization

- Both production and logistics optimization
- 10 factories, each with several production lines
- Large number of stores and distribution centers

A challenging context for LocalSolver

- 20,000,000 expressions including 3 million binaries
- Rich model involving setup costs, delivery times, packaging, etc.
- Vain attempts to solve the problem with MIP solvers
- LocalSolver finds high-quality solutions in 5 minutes



Application panorama

TV media planning

Outdoor & indoor advertising

Logistic clustering and routing

Road maintenance planning

Network deployment planning

Loan assembling optimization

Placement of nuclear fuel assemblies in pools

Airline network management

Weapon resource allocation

Packing and transportation of military equipment

Where LocalSolver goes?

Novelties in version 5.5

Set-based modeling

Structured decisional operator list(n)

- Order a subset of values in domain {0, …, n-1}
- Each value is unique in the list

Classical operators to interact with "list"

- count(u): number of values selected in the list
- get(u,i) or u[i]: value at index i in the list
- indexOf(u,v): index of value v in the list
- contains(u,v): equivalent to "indexOf(u,v) != -1"
- disjoint(u1, u2, …, uk): true if u1, u2, …, uk are pairwise disjoint
- partition(u1, u2, …, uk): true if u1, u2, …, uk induce a partition of {0, …, n-1}

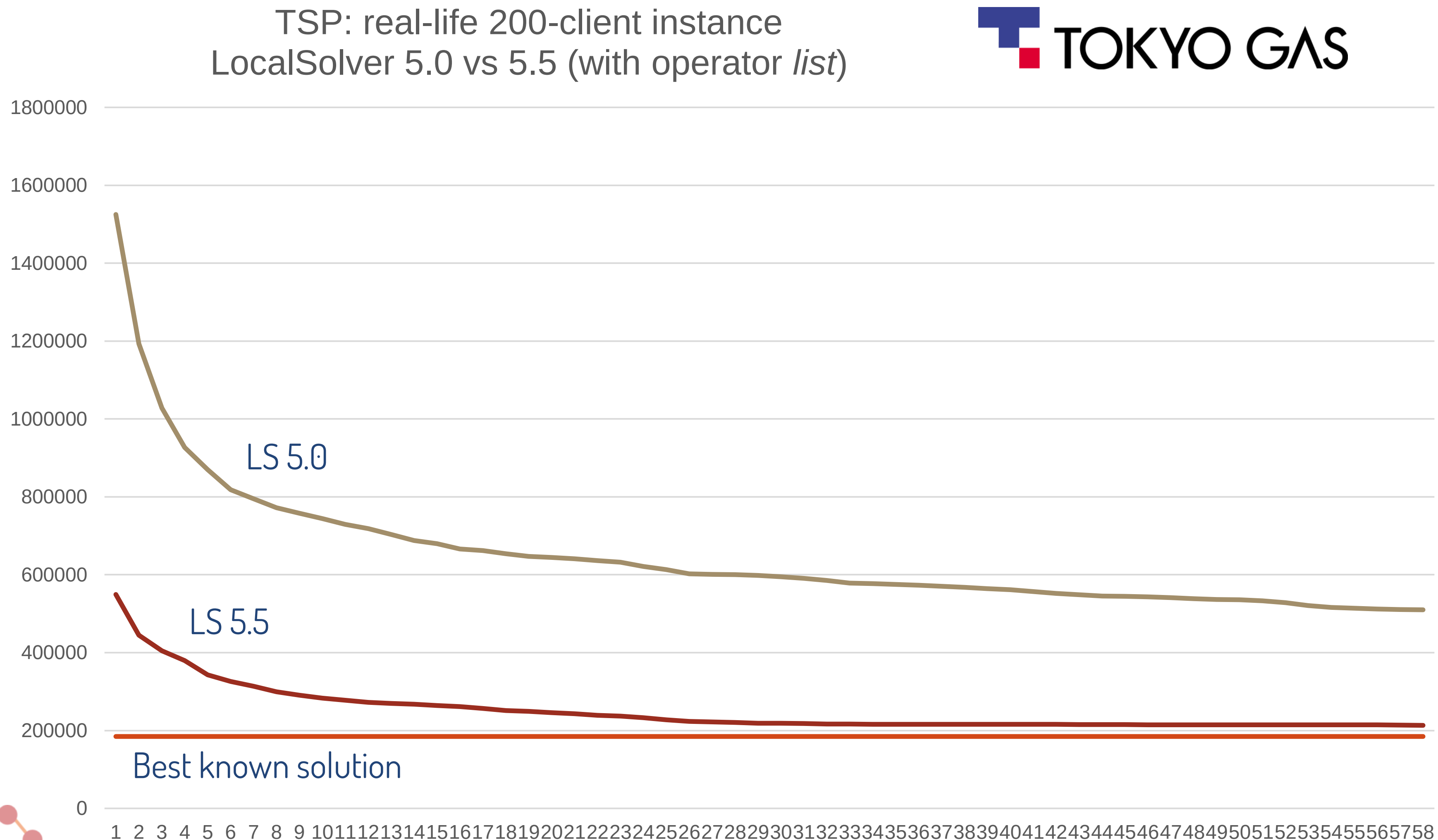
Traveling salesman

```
function model() {
  x <- list(N) ; // order n cities {0, ..., n-1} to visit
  constraint count(x) == N; // exactly n cities to visit
  minimize sum[i in 1..N-1]( Dist[ x[i-1] ][ x[i] ] )
                            + Dist[ x[N-1] ][ x[0] ] ; // minimize sum of traveled distances
}
```

Could you imagine a simpler model?

- Natural declarative model: straightforward to understand
- Common set-oriented concepts: easy to learn
- Even easier for people with basic programming skills
- Compact: linear in the size of input → highly scalable

Traveling salesman



Vehicle routing

```
function model() {
  x[1..K] <- list(N) ; // for each truck, order the clients to visit
  constraint partition( x[1..K] ); // each client is visited once
  distances[k in 1..K] <- sum[i in 1..N-1]( dist( x[k][i-1], x[k][i]) )
              + dist( x[k][N-1], x[k][0] ); // traveled distance for each truck

  minimize sum[k in 1..K]( distances[k] ); // minimize total traveled distance
}
```

To go further, to make it simpler

- Sets (unordered) versus lists (ordered)
- Collections of objects instead of values
- Ability to iterate and project over collections (lambda expressions)

CVRP benchmarks

CVRP - instances A

- 32 to 80 clients, 10 trucks max.
- 27 instances
- 5 minutes of running time
- LS binary: 3 % avg. opt. gap
- LS list: 1 % avg. opt. gap

CVRP - instances X100-500

- 100 to 500 clients, 138 trucks max.
- 67 instances
- 5 minutes of running time
- LS binary: N/A
- LS list: 9 % avg. opt. gap

CVRPTW benchmarks

CVRPTW – instances Solomon R100

- 101 to 112 clients, 19 trucks max.
- 13 instances
- 5 minutes of running time
- LS binary: N/A
- LS list: 3 % avg. opt. gap

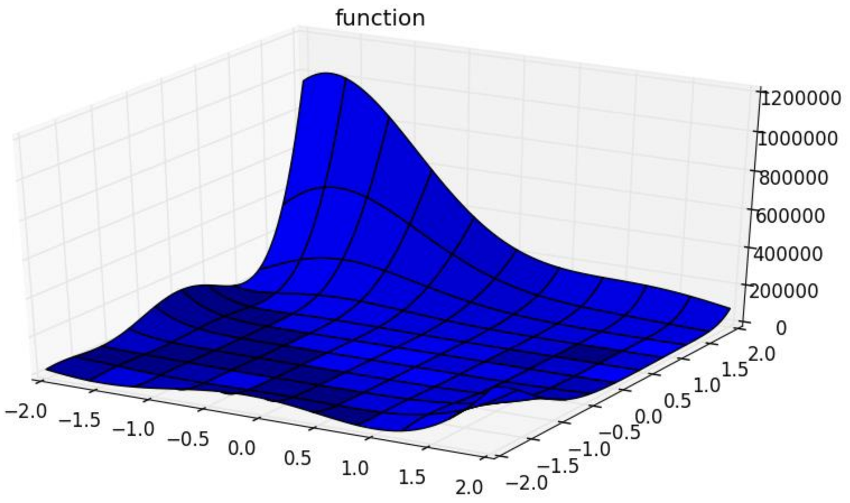
CVRPTW – instances Solomon R200

- 201 to 208 clients, 4 trucks max.
- 8 instances
- 5 minutes of running time
- LS binary: N/A
- LS list: 8 % avg. opt. gap

Black-box optimization

Context

- Unknown objective (oracle)
- Costly to evaluate
- Derivative-free
- Continuous & integer decisions
- Bounds on decisions



Many applications in engineering

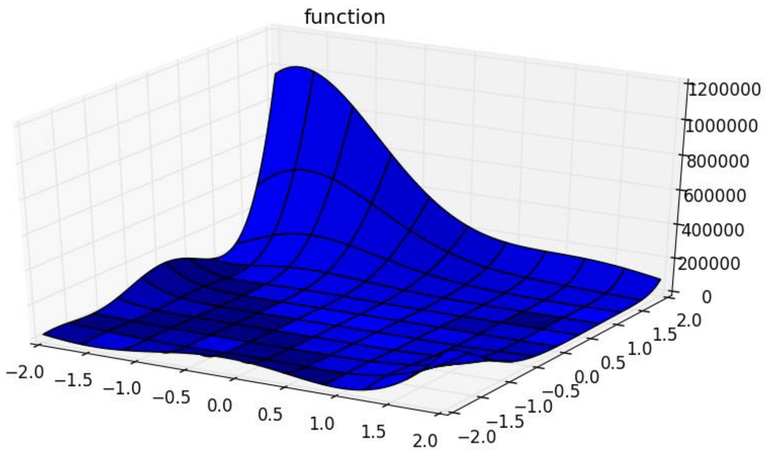
- Multidisciplinary/parametric optimization
- Simulation optimization (noisy, nondeterministic)

→ Design optimization of materials/systems: mechanics, electricity, logistics, etc.

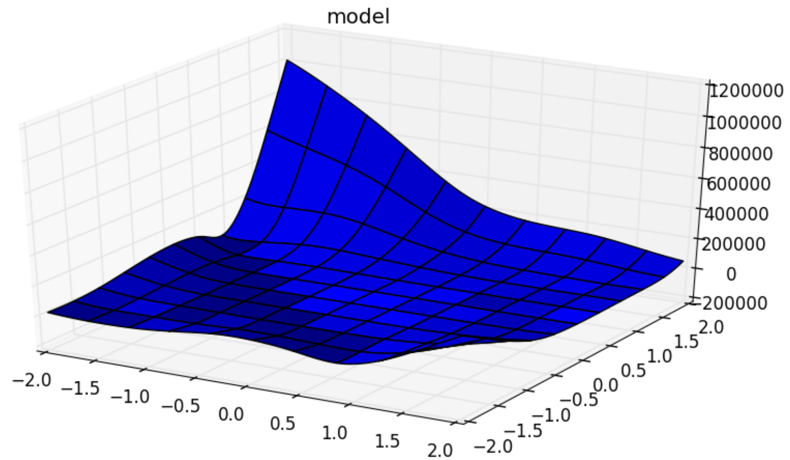
Learning

Learn the objective function landscape

- Objective landscape modeled by Radial Basis Functions
- Several models are built with different techniques/parameters
- Automatic selection of the most promising models for optimization



Objective Function



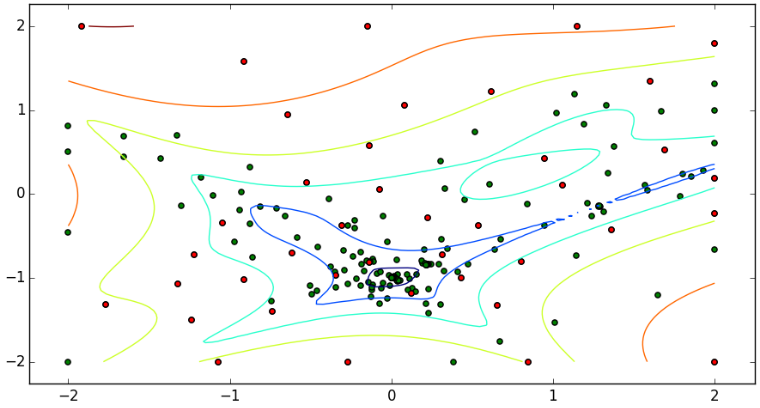
Objective Model

Optimization

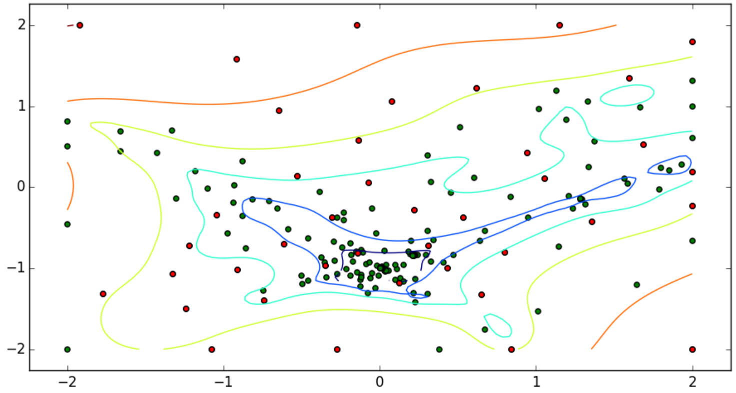
Exploitation & diversification

- Exploitation: optimize over the objective model
- Diversification: explore new promising regions

→ NLP subproblems solved through LocalSolver techniques:
local & direct search, gradient-based line search, etc.



Objective Function



Objective Model

Benchmark

Instances

- 25 instances from the recent paper by Costa and Nannicini.

  RBFOpt: an open-source library for black-box optimization with costly function evaluations. Optimization Online. (under review)
- 20 runs per instance, 150 calls max. to the black-box per run
- Numerical precision: 1e-6

Preliminary results

- RBFOpt: 345 opt. solutions found, 82 calls avg. per run
- LocalSolver: 310 opt. solutions found, 94 calls avg. per run
- NOMAD (GERAD): 170 opt. solutions found

Benchmark

	LocalSolver			RBFOpt			NOMAD	
Instance	**#sol**	**Avg. Eval**	**Error (%)**	**#sol**	**Avg. Eval**	**Error (%)**	**#sol**	**Error (%)**
branin	20	23	0,0	20	31	0,0	20	0,0
camel	20	26	0,0	20	34	0,0	19	4,0
ex_4_1_1	20	11	0,0	20	14	0,0	20	0,0
ex_4_1_2	20	51	0,0	20	9	0,0	20	0,0
ex_8_1_1	20	10	0,0	20	7	0,0	19	2,5
ex_8_1_4	20	44	0,0	20	25	0,0	0	341,5
gear	20	34	0,0	20	7	0,0	0	388,0
goldsteinprice	18	122	0,1	20	53	0,0	16	450,0
hartman3	8	130	1,2	20	45	0,0	15	9,4
hartman6	8	121	11,0	17	101	5,1	0	5,7
least	0	150	1308,0	0	150	204,7	0	129,0
nvs04	20	70	0,0	19	64	194,4	4	9997,0
nvs06	16	127	1,0	0	150	13,3	9	8,7
nvs09	20	15	0,0	20	14	0,0	16	1,2
nvs16	8	138	949,0	20	49	0,0	9	885,0
perm0_8	0	150	109,0	0	150	147,2	0	412,0
perm_6	0	150	2424958,0	0	150	44134,7	0	311032,0
rbrock	20	83	0,0	5	136	10,8	0	43,2
schoen_10_1	4	145	66,7	11	139	28,8	0	119,5
schoen_10_2	0	150	96,2	14	133	1,6	0	115,7
schoen_6_1	18	101	100,8	18	101	1,8	0	51,5
schoen_6_2	10	120	28,0	16	102	32,7	0	54,2
shekel10	8	118	29,6	13	107	60,1	0	56,9
shekel5	6	127	51,6	7	126	51,7	1	46,1
shekel7	6	127	28,5	5	137	47,0	2	47,9
	310			345			170	

Roadmap

John N. Hooker (2007)

“Good and Bad Futures for Constraint Programming (and Operations Research)”
Constraint Programming Letters 1, pp. 21-32

“Since modeling is the master and computation the servant, no computational method should presume to have its own solver.

This means there should be no CP solvers, no MIP solvers, and no SAT solvers. All of these techniques should be available in a single system to solve the model at hand.

They should seamlessly combine to exploit problem structure. Exact methods should evolve gracefully into inexact and heuristic methods as the problem scales up.”

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